Test 1 study sheet

The parabola

$y^2 = 4px \qquad e = 1$

Ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad b^2 = a^2(1-e^2) \qquad c = ae \qquad k = \frac{a}{e} \qquad 0 < e < 1$

Hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad b^2 = a^2(e^2-1) \qquad c = ae \qquad k = \frac{a}{e} \qquad e > 1 \qquad y = \pm\frac{b}{a}x \leftarrow$ asymptotes

Polar coordinates

$x = r\cos\theta \quad y = r\sin\theta \qquad (r, \theta)$

line: $r = \frac{\theta}{\cos(\theta - \theta_0)}$ conic: $r = \frac{e\partial}{1 + e\cos(\theta - \theta_0)}$ circle: $r = 2a\cos\theta$

Special graphs:

Limacon: $r = a \pm b\cos\theta$ if $a = b$ the graph is a cardioid

Rose: $r = a\cos n\theta \quad r = a\sin n\theta$ If n is even, there are $2n$ leaves

leaves are of length a. If n is odd, there are n leaves

Area:

$A = \int_a^b \frac{1}{2} f(\theta)^2 \, d\theta$

Cycloid: $x = a(t - \sin t)$

$y = a(1 - \cos t)$

Length:

$L = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2} \, dt$

Vector dot product:

$\bar{U} \cdot \bar{V} = |\bar{U}||\bar{V}|\cos\theta = U_x V_x + U_y V_y$

Vector magnitude:

$|\bar{U}| = \sqrt{u_x^2 + u_y^2}$

Vector projections:

$Pr_{\bar{V}}\bar{U} = \left(\frac{\bar{U} \cdot \bar{V}}{\bar{V} \cdot \bar{V}}\right)\bar{V}$

Vectors:

All vectors can be represented as $F(t) = xi + yj = f(t)i + g(t)j$

Curvilinear motion

$r(t) = f(t)i + g(t)j$

$v(t) = r'(t) = f'(t)i + g'(t)j$

$a(t) = v'(t) = r''(t) = f''(t) + g''(t)$

Unit tangent vector:

$\bar{T}(t) = \dfrac{r'(t)}{|r'(t)|} = \dfrac{v(t)}{|v(t)|}$

Unit normal vector:

$\bar{N}(t) = \dfrac{\bar{T}'(t)}{|\bar{T}'(t)|}$

EBX

ECX - loop ctr

ESP - stack ptr

ESP

④ Ampere-Maxwell law $\oint_{path\ \Gamma} B \cdot d\Gamma = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt} \int_{surface\ whose} E \cdot dA$

path Γ

Test 2 study sheet

Graphing in $\mathbb{R}^3$ 1) find the traces in the 3 planes
2) look for symmetry
3) look at the intersection w/ any plane

Vectors in $\mathbb{R}^3$ $\quad \bar{u} = ai + bj + ck \quad |\bar{u}| = \sqrt{a^2+b^2+c^2}$

Planes in $\mathbb{R}^3$ $\quad A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ $\quad$ Distance to point: $\dfrac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}$

$D = -Ax_0 + By_0 + Cz_0$

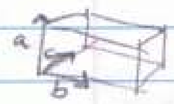
Dot product: $\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta = u_1v_1 + u_2v_2 + u_3v_3$

Cross product: $\bar{u}\times\bar{v} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} i - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} j + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} k = |\bar{u}||\bar{v}|\sin\theta$ $\quad i \times j = k$

$\bar{u}\times\bar{v} = -(\bar{v}\times\bar{u}) \quad c(u\times v) = cu\times v = u\times cv \quad (u\times v)\cdot w = (v\times w)\cdot u = (w\times u)\cdot v$

$(u\times v)\times w = (u\cdot w)v - (u\cdot v)w$

Area of Parallelogram  $A = \bar{u}\times\bar{v}$

Volume of parallelepiped $\quad V = |\bar{b}\times\bar{c}||\bar{a}|\cos\theta = \bar{a}\cdot(\bar{b}\times\bar{c})$ 

Curves in $\mathbb{R}^3$ $\quad \bar{r}(t) = f(t)i + g(t)j + h(t)k \quad T(t) = \dfrac{\bar{r}'(t)}{|\bar{r}'(t)|} \quad N(t) = \dfrac{\bar{T}'(t)}{|\bar{T}'(t)|}$

$v(t) = r'(t) \quad a(t) = v'(t)$

Line $\quad x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Cylindrical coordinates

$(x,y,z) \Rightarrow (r,\theta,z) \quad x = r\cos\theta \quad y = r\sin\theta \quad z = z$

$r = x^2 + y^2$

Spherical coordinates

$(x,y,z) \Rightarrow (\rho,\theta,\phi) \quad x = \rho\cos\theta\sin\phi \quad \rho = \sqrt{x^2+y^2+z^2}$

$y = \rho\sin\theta\sin\phi$

$z = \rho\cos\phi$

functions of several variables

$f(x,y)$ Natural Domain: largest set that function can be evaluated

Level curves

$f(x,y) = c$ solve equation and trace curve on x,y plane

Calc III

Test 3 study material

Partial Derivatives

1st order

$f_x = \frac{\partial F}{\partial x}$ partial derivative w/ respect to 'x', any 'y' that appears should be handled as a constant

$f_y = \frac{\partial F}{\partial y}$ any x that appears should be handled as a constant

2nd order

$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$ $f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$

$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)$ $f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$

Limits

$\lim\limits_{(x,y)\to(x_0,y_0)} f(x) = \ldots$

Differentiability

Remember:

1) $f(p+h) = f(p) + \nabla f \cdot h + \varepsilon(h)\cdot h$
2) $p = \langle a, b \rangle$ $h = \langle h_1, h_2 \rangle$
3) $\nabla f(p) = \langle f_x(p), f_y(p) \rangle$ ← known as the gradient of F
4) $\varepsilon(h) = \langle \varepsilon_1(h_1, h_2), \varepsilon_2(h_1, h_2) \rangle$ $\lim\limits_{h\to 0} \varepsilon(h) = 0$

Directional Derivative

The directional derivative of f at p in the direction of unit vector $\bar{u}$ is:

$$D_u f(p) = \bar{u} \cdot \nabla f(p)$$

Rates of change

Direction of fastest increase is: $\bar{u} = \frac{\nabla f(p)}{|\nabla f(p)|}$

" " fastest decrease is: $-\bar{u}$

The chain rule

1) $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}$ where $z = f(x, y)$, $x = x(t)$, $y = g(t)$

2) $\dfrac{\partial z}{\partial t} = \dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}$ where $z = f(x, y)$, $x = x(s, t)$, $y = y(s, t)$

Tangent Planes

The tangent plane to the surface $F(x, y, z) = K$ at $P(x_0, y_0, z_0)$ is

$$F_x(p)(x - x_0) + F_y(p)(y - y_0) + F_z(p)(z - z_0) = 0$$

Maxima/Minima

$D(p) = f_{xx} \cdot f_{yy} - f_{xy}^2$

1) If $D(p) > 0$ & $f_{xx}(p) < 0$ p is a local max
2) If $D(p) > 0$ & $f_{xx}(p) > 0$ p is a local min
3) If $D(p) < 0$ p is a saddle pt
4) $D(p) = 0$ gives no info

Implicit Differentiation

$$\frac{\partial y}{\partial x} = \frac{dy}{dx} = \frac{-\partial F/\partial x}{\partial F/\partial y}$$

calc III

Test 4 study sheet

*Note for integrals of any order, its important to sketch the Domain in order to get the region of integration(s)

Lagrange Multipliers

- To find max / min

$f_x = \lambda g_x$

$f_y = \lambda g_y$

$g(x,y) = k$

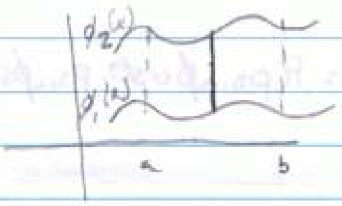
solve for x & y then compare values

*** Double Integrals ***

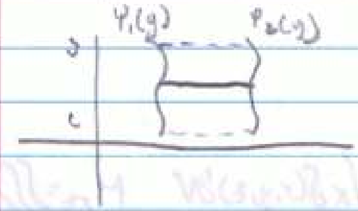
$\iint_R f(x,y)\,\partial A$ used for many purposes, mainly to find volume under surface in 3 space

integral find area
integral find volume
integral find...

In order to find R look at Domain



x-simple $\iint_R f(x,y)\,\partial A = \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} f(x,y)\,\partial y\,\partial x$



y-simple $\iint f(x,y)\,\partial A = \int_c^d \int_{\psi_1(y)}^{\psi_2(y)} f(x,y)\,\partial x\,\partial y$

Double integrals using Polar coordinates

$x \to r\cos\theta$ $\qquad x^2+y^2=r^2$

$y \to r\sin\theta$

$\partial A \to r\,\partial r\,\partial\theta$

Applications of Double integral

$\delta(x,y)$ density function

Mass: $m = \iint_S \delta(x,y)\,\partial A$

centers of mass: $\bar{x} = \iint_S x\,\delta(x,y)\,\partial A \Big/ \iint_S \delta(x,y)\,\partial A$

$\bar{y} = \iint_S y\,\delta(x,y)\,\partial A \Big/ \iint_S \delta(x,y)\,\partial A$

Moments of Inertia:

$I_x = \iint_S y^2\,\delta(x,y)\,\partial A$ $\quad I_y = \iint_S x^2\,\delta(x,y)\,\partial A$

$I_z = I_x + I_y = \iint (x^2+y^2)\,\delta(x,y)\,\partial A$

surface area

$$A(G) = \iint_S \sqrt{f_x^2 + f_y^2 + 1}\, \partial A$$

Triple Integrals

$$\iiint_S f(x,y,z)\, \partial V = \iint_R \left[\int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z)\, \partial z \right] \partial A$$

In cylindrical:

$$\partial V = \partial z\, \partial A = \partial z\, r\, \partial r\, \partial \theta \qquad f(x,y,z) = f(r\cos\theta, r\sin\theta, z)$$

In spherical:

$$\partial V = \rho^2 \sin\phi\, \partial \rho\, \partial \theta\, \partial \phi \qquad f(x,y,z) = f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$$

Applications:

Mass: $m = \iiint_S \delta(x,y,z)\, \partial V$

Moment: $M_{xy} = \iiint_S z\,\delta(x,y,z)\,\partial V \qquad M_{yz} = \iiint_S x\,\delta(x,y,z)\,\partial V \qquad M_{xz} = \iiint_S y\,\delta(x,y,z)\,\partial V$

center of mass: $\bar{z} = \dfrac{M_{xy}}{m} \qquad \bar{x} = \dfrac{M_{yz}}{m} \qquad \bar{y} = \dfrac{M_{xz}}{m}$

1. hw
2. finish sheet
3. study sheets
   fall material
   review book