Phy 212 - Test I study sheet:

Equations:

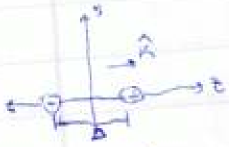
$$F_{elec} = \frac{1}{4\pi\varepsilon_0}\frac{|q||Q|}{r^2}$$ for two pointlike charges

$$\vec{F}_{elec} = q\vec{E}$$

dipole moment: $\vec{p} = |Q|\,d$ $\qquad \tau_{on\ dipole} = \vec{p}\times\vec{E}$

Electric field.

Charge Distribution	E-field
Pointlike charge	$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$
Dipole along axis ($z \gg d$)	$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{z^3}\hat{k}$
Uniformly charged ring (on axis)	$E = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}\hat{k}$
uniform charged disk (on axis)	$E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}\left[1-\frac{z}{(z^2+R^2)^{1/2}}\right]\hat{k}$
uniformly charged infinite sheet	$E = \sigma/2\varepsilon_0$
2 infinitely charged uniform sheets w/ opposite charges	$E = \sigma/\varepsilon_0$
uniformly charged spherical shell	inside ($r<R$) $E = 0$; outside ($r>R$) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$
Uniformly charged sphere	inside ($r<R$) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r\,\hat{r}$; outside ($r>R$) $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
Along infinitely straight line:	$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}\hat{r}$



Electric flux: $\Phi = \int \vec{E}\cdot d\vec{S} = \frac{Q_{total\ charge\ within\ object}}{\varepsilon_0}$

$W_{elect\ force} = -\Delta PE$

voltage: $V = \frac{PE\ of\ q}{q}$ $\qquad \Delta V = -\int \vec{E}\cdot d\vec{r}$ change in electric potential (V) between 2 pts.

$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ for a point like charge

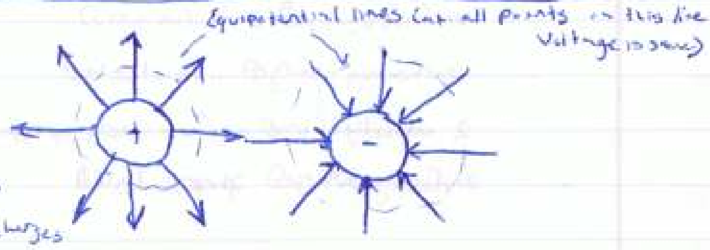
Electric potentials (voltages) of various charge distributions:

Charge Distribution	Electric Potential
Two infinite sheets, oppositely charged, uniform E-field	$V(x) - V(0) = -E(x)$
Pointlike charge	$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
Uniform charged ring, distance z along axis	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(z+R^2)^{1/2}}$
Uniform charged disk, distance z along axis	$V = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}\left[(z^2+R^2)^{1/2} - z\right]$
Uniformly charged sphere, $r > R$	$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$
Uniformly charged conducting sphere $r < R$	$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ (constant)
Uniformly charged insulating sphere $r < R$	$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}\left[3 - \frac{r^2}{R^2}\right]$

Concepts

~~static~~ charges:

E-field points away from + charges, & towards − charges



For non-static charges placed in E-fields:

+ charges experience force parallel to field, − charges experience force antiparallel to field.

The electron volt: $1\,eV = 1.602 \times 10^{-19}\,J$

Phy 212 Test 2 Study Sheet

Energy Conservation $(KE+PE)_i = (KE+PE)_f$ $KE = \frac{1}{2}mv^2$ $PE = qV$

Dipole in uniform electric field: moment: $\bar{p} = q\bar{d}$ torque: $\bar{\tau} = \bar{p} \times \bar{d}$ (d - distance between charges)

E-field points from higher potential (+) to lower (−)

~~Capacitors~~

PE of several charges:  $PE = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{d_1} + \frac{q_1 q_3}{d_3} + \frac{q_2 q_3}{d_2} + \dots\right)$

Capacitors:

* capacitance: $C = \left|\frac{q}{V}\right| = \frac{\varepsilon A}{d}$

Energy stored within: $PE = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}|Q||V| = \frac{1}{2}CV^2$

Potential energy density for any electric field:

* $\frac{PE}{Ad} = \frac{1}{2}\varepsilon E^2$

* In these equations $\varepsilon = K\varepsilon_0$ where K is the dielectric constant, if dielectric material is present. If not $\varepsilon = \varepsilon_0$

Ohm's law

$V = IR$

* $I = n q \overset{\text{velocity of charge}}{v} \underset{\text{cross sectional area of pipe}}{S}$ for a plane surface

n - # charges/unit volume, q - charge of each

for non-plane surfaces $j = nqv$ and $I = \int j$

* $R = \frac{\rho l}{A}$ ρ = resistivity l - length of medium A - crossectional area of medium

Power $P = IV = \frac{V^2}{R}$

AC circuits

$V(t) = V_{max}\sin(\omega t)$ $\omega = 2\pi f$ f = frequency

$P(t) = \left[\frac{V_{max}^2}{R}\right]\sin^2(\omega t)$

$I(t) = \left[\frac{V_{max}}{R}\right]\sin(\omega t)$

KCL

currents into node = currents out

KVL

sum of voltage rises = sum of voltage drops in a loop

Connections

	Series	Parallel
Capacitors	$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$	$C_T = C_1 + C_2 + C_3 + \dots$
Resistors	$R_T = R_1 + R_2 + R_3 + \dots$	$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

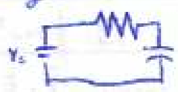
Resistance thermometers: $\rho = \rho_0 [1 + \alpha(T - T_0)]$ α - temperature coefficient of resistivity
(Don't need to know) $R = R_0 [1 + \alpha (T - T_0)]$ - constant based on material

Real batteries: $I = \frac{V_0}{R_T + r}$ $V_{term} = V_0 \frac{R_T}{R_T + r}$ R_T - resistors outside battery
r - battery internal resistance

As a battery is used $r \to \infty$ so I and $V_{term} \to 0$

Charging a capacitor

For the following circuit



$I(t) = \frac{V_0}{R} e^{-t/RC}$ when charging and $-I(t)$ when discharging

Vectors review

$V \cdot W = V_x W_x + V_y W_y + V_z W_z = |V||W| \cos\theta$

$V \times W = (V_y W_z - V_z W_y) i - (V_x W_z - V_z W_x) j + (V_x W_y - V_y W_x) K$

$|V \times W| = |V||W| \sin\theta$

Right hand rule:

$V \times W$ produces a vector perpendicular to both. Direction taken by pointing fingers of right hand towards V, & curling towards W. Point thumb perpendicular to hand to obtain direction of new vector

$i \times i = j \times j = k \times k = 0$ $(i \times j) = -(j \times i) = K$ $(j \times k) = -(k \times j) = i$ $(k \times i) = -(i \times k) = j$

Matrices

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} e & f \\ h & i \end{vmatrix}$$

$$= a(ei - hf) - b(di - gf) + c(ei - hf)$$

Old material:

pt like chg $\rightarrow$ $F_{elec} = \frac{1}{4\pi\varepsilon_0} \frac{|q||Q|}{r^2} = q\vec{E}$ Electric flux $= \Phi = \int \vec{E} \cdot \vec{ds} = \frac{Q_{total\ within}}{\varepsilon_0}$

$W_{elec} = -\Delta PE$ $V = \frac{PE_{elec}}{q}$ $\Delta V = -\int E \cdot dr$

$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$ for pt like chrg. E fields point away from + charges, towards −

$K = \frac{1}{4\pi\varepsilon_0}$

Geometry	E field $E =$	$V(r) =$
Dipole on axis	$K \frac{2p}{z^3}$	
Uniform charged ring	$K \frac{zq}{(z^2 + R^2)^{3/2}}$	$K \frac{Q}{(z^2 + R^2)^{1/2}}$
Uniform charged disk	$K \frac{2Q}{R^2} \left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right]$	$K \frac{2Q}{R^2} \left[(z^2 + R^2)^{1/2} - z\right]$
Uniformly charged infinite sheet	$E = \sigma / (2\varepsilon_0)$	
2 uniformly charged infinite sheets w/ opposite charges	$E = \sigma / \varepsilon_0$	$V(x) - V(0) = -E(x)$
Uniformly charged shell, sphere	inside $E = 0$	conducting inside: $K \frac{Q}{R}$
Uniformly charged sphere (solid)	inside: $K \frac{Q}{R^3} r$	insulating: inside: $K \frac{Q}{2R} \left[3 - \frac{r^2}{R^2}\right]$
Along infinite straight line	$K \frac{2\lambda}{r}$	

outside treat as pt. charges

Notes: z refers to a distance along the perpendicular axis. R refers to radius of obj
r - refers to distance away from object

Phy 212 - Exam 3 Test Sheet

Maxwell's equations - summarize properties of electric & magnetic fields due to sources.

- We found that a changing electric field produces a perpendicular magnetic field & vice-versa.

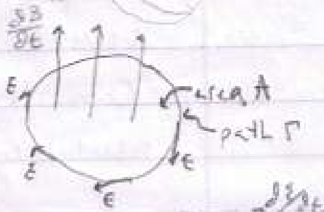
1) Gauses law for E ⇒ $\Phi_{elect} = \int_{\text{closed surface}} E \cdot \partial A = \frac{Q_{net\ within}}{\varepsilon_0}$

2) Gauses law for B ⇒ $\Phi_{magnetic} = \int_{\text{closed surface}} B \cdot \partial A = 0$

- around a loop: $\Phi_{mag} = B \cdot A$

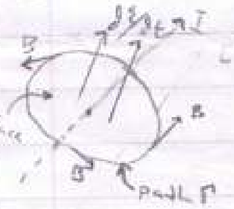
3) Faraday's law ⇒ Induced emf $= \oint_{\text{path } \Gamma} E \cdot \partial l = -\frac{\partial \Phi_{mag}}{\partial t} = -\frac{\partial}{\partial t} \int_{\text{surface bounded by } \Gamma} B \cdot \partial A$

Induced emf is a voltage



4) Ampere-Maxwell Law ⇒ $\oint_{\text{path } \Gamma} B \cdot \partial r = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int_{\text{surface whose boundary is } \Gamma} E \cdot \partial A$

This expression minus the stuff to the right of the + is called amperes law



Given a loop of wire in a magnetic field, the current in the loop will always try to oppose the change in the magnetic field in accordance w/ the RHR
e.g. point thumb in direction B-field is getting weaker and curl fingers indicating direction of I.

Right Hand Rule (RHR) applications:

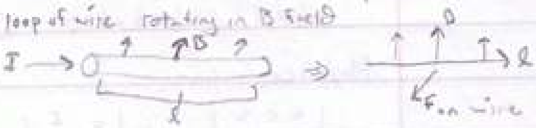
$A = B \times C$
1) point fingers of right hand in direction of B
2) curl towards C
3) thumb is pointing in direction of A

$F_{mag} = qV \times B$ - Given a V & B, -q will experience a force opposite of +q

$\tau = \mu \times B$ where $\mu = nIA$ torque of loop of wire rotating in B field

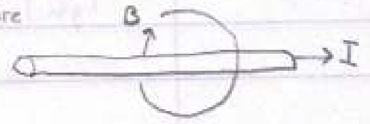
pot energy: $PE = -\mu \cdot B$

$F = I l \times B$ along a long wire



A current in a wire I will generate a B-field circling the wire

to establish a relationship use Ampere-Maxwell



Inductors - store magnetic energy. Are to current what capacitors are to voltage - a coil of wire

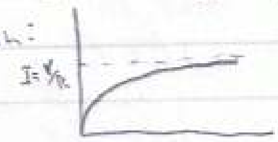
Time constant $\tau = \frac{\text{Inductance}}{\text{Resistance}} = \frac{L}{R}$

$$I_i(t) = I - Ie^{-t/\tau} = \frac{V_s}{R_{eq}}\left(1 - e^{-t/\tau}\right)$$

Inductance $L = \frac{\Phi_{mag}}{I}$ & $V_{coil} = L\frac{\partial I}{\partial t}$

Inductors in series/parallel can be summed like resistors

Charging graph:



Theoretically an inductor never fully gets magnetized. Though we say it does after 5τ

Energy stored within $U = \frac{1}{2}LI^2$ Power absorbed $P = IL\frac{\partial I}{\partial t}$

If a conductor is magnetized and placed in a circuit w/ no sources it will act as a current provider old material

$W_{\text{elec force}} = -\Delta PE$ $\qquad \vec{F}_{elec} = q\vec{E}$

$V = \frac{PE_{\text{of } q}}{q}$ $\qquad$ Pointlike charges: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

electric dipole moment: $\vec{p} = |Q|\vec{d}$ $\qquad \vec{\tau}_{\text{dipole}} = \vec{p} \times \vec{E}$

Pointlike charge $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ $\qquad$ E-field points away from + charges, towards -

$1\,eV = 1.602 \times 10^{-19}\,J$

$\tau = RC$

Capacitance $C = \left|\frac{q}{V}\right| = \frac{\varepsilon A}{d}$ $\qquad$ Energy stored $PE = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}|Q||\Delta V| = \frac{1}{2}CV^2$

Potential energy density for electric field

$$\frac{PE}{Ad} = \frac{1}{2}\epsilon E^2$$

Circuitry

$V = IR \qquad I = nqvS \qquad R = \frac{\rho l}{A} \qquad P = IV$

$\omega = 2\pi f$

Vectors:

$$V \cdot W = V_x W_x + V_y W_y + V_z W_z = |V||W|\cos\theta$$

$$V \times W = (V_y W_z - V_z W_y)i - (V_x W_z - V_z W_x)j + (V_x W_y - V_y W_x)k$$

$$= |V||W|\sin\theta$$

$i \times i = j \times j = k \times k = 0 \qquad (i \times j) = -(j \times i) = k$ and so on ...

Matrices

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} e & f \\ h & i \end{vmatrix}$$

$$= a(ei - hf) - b(di - gf) + c(ei - hf)$$

$k\left(\frac{q_1 q_2}{r^2}\right)R$ $\quad$ $k\left(\frac{q_1}{r^2}\right)R$

$W = -\Delta PE = F(\text{displacement}) = E_{field}(q)(\text{displacement}) = -\Delta V q$

$V(r) = \Delta V r + C$

$y(x) = mx + B$

$-\Delta PE = E_{field}(r)(q) = -\Delta V q$

$E_{field}(r) = -\Delta V$

(E_{field} = -Derivative of V in terms of r)

$(E_{field})(q) = F$

(Graph of E is multiplied by a constant (q))

PE

→ The graph of E_{field} = Slope of the graph of V w. r

$PE = Vq$

Potential Energy

(graph of V, multiplied by a Constant (either Stretched (if $q > 1$) Compressed (if $q < 1$) or the same (if $q = 1$) if ($q = 0$, no potential & No PE)

F(displacement) = Work

(Graph of F is multiplied by a constant (displacement))

$-\Delta PE = \text{Work}$

(Work is derivative of Potential Energy)

Hall Effect: $V_{Hall} = \frac{I\,?}{nq\partial} = E\ell$

Electricity: $P = I^2 R$

$R = \frac{\rho \ell}{A}$ ← resistivity

$A = 2\pi r h$ for wires

Capacitors:

$C = Q/V$

$C = \frac{\varepsilon_0 A}{d}$

wire: $I = qnAV$ ← velocity

↳ charge density

Electric motor:

$V(t) = NBA \cdot \omega \sin(\omega t)$

note: $\omega = 2\pi f$

$I = \frac{\partial Q}{\partial t}$

Work to charge a capacitor:

$PE = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$

Electric flux: $\Phi = \frac{Q}{\varepsilon_0}$

Magnetic flux: $\Phi = BA\cos(\omega t)$

Electric fields

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

pointlike

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

uniformly charged sphere

$(r<R)\ E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}$

$(r>R)\ E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

uniformly charged ring

$E = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$

uniformly charged disk

$E = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]$

uniform sphere shell

$(r<R)\ E = 0$

$(r>R)\ E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

uniform inf sheet

$E = \frac{\sigma}{2\varepsilon_0}$

2 uniform inf sheets w/ opp charges $E = \frac{\sigma}{\varepsilon_0}$

Voltages

$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$(r>R)\ V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$(r<R)$ conducting

$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

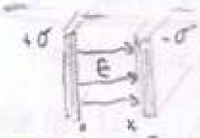
$(r<R)$ insulating

$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$

Distance z along axis

$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(z^2+R^2)^{1/2}}$

$V = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{R^2}\left[(z^2+R^2)^{1/2} - z\right]$



$V(x) - V(0) = -Ex$

Relativity:

$T = \gamma T_0 \qquad \lambda = \frac{\ell_0}{\gamma}$

General Equations:

$U_x' = \frac{U_x - V}{1 - \frac{VU_x}{c^2}} \qquad U_x = \frac{U_x' + V}{1 + \frac{VU_x'}{c^2}}$

Lorentz Transformations:

$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$

$y' = y \qquad y = y'$

$z' = z \qquad z = z'$

$t' = \gamma\left(t - \frac{v}{c^2}x\right) \qquad t = \gamma\left(t' + \frac{v}{c^2}x\right)$

for slow speeds:

$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

Relativistic momentum:

$p = \gamma m v$

$p = \frac{v}{c^2}E$

$E^2 = p^2c^2 + m^2c^4$

Total Relativistic Energy:

$E = KE + mc^2$

$E = \gamma mc^2$

Relativistic KE:

$KE = (\gamma - 1)mc^2$

$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$

$\boxed{\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$

Waves:

$Y(x,t) = A\cos[kx - \omega t]$

↑ indicates direction of wave: right −, left +

$V = \lambda f$

$P = \frac{1}{f} = T$

$K = \frac{2\pi}{\lambda}$

↑ wave vector ↑ wavelength

$\omega = 2\pi f$

Hz = rev/sec

λ = meters

Random electrical equations:

$W_{elec} = -\Delta PE \qquad V = \frac{PE_{elec}}{q} \qquad E = vB$

$\tau = p \times E$

$F_{elec} = qE$ ← similar to gravity

$B = \frac{\mu_0 I}{2\pi d}$

force of diff. particle: $F_e = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$

$F_m = qv \times B$

$L = \frac{\Phi_{mag}}{I}$ ← inductor

$B = \frac{\mu_0 I}{2\pi d}$

$p = |Q|d$ ← dipole moment

$\tau = \mu \times B$ ← torque

$F = I\ell \times B$ ← force on current carrying wire

$\mu = AI$

$PE = -\mu \cdot B$

Area of cylinder = $2\pi r h$

Temp coeff. of resistivity:

$\rho = \rho_0 [1 + \alpha (T - T_0)]$

$R = R_0 [1 + \alpha (T - T_0)]$

Charging a capacitor:

$I(t) = I_0 e^{-t/RC}$

$V_0 = \frac{E}{B}$ velocity of a charged

Voltage between two plates (capacitor):

$V = \varepsilon \partial$

Dot (scalar) product

$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$

where θ is the ∠ between A & B

$|A| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

cross product

$|A \times B| = |A||B| \sin\theta$

$A \times B = (A_y B_z - B_y A_z) i - (A_x B_z - B_x A_z) j + (A_x B_y - B_x A_y) j$

Common equation

F=ma

p=mv

v=rω

Around a closed path loop:

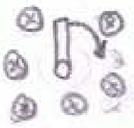
$\frac{W_{elec}}{q} = \varepsilon_{induced}(2\pi r)$ & $\varepsilon_{induced} = -\frac{r}{2} \frac{\partial B}{\partial t}$

Current always tries to oppose change in magnetic field

Energy density assoc w/ electric field: $\frac{1}{2}\frac{B^2}{\mu_0}$

$\varepsilon = \frac{\sigma}{\varepsilon_0}$ - electric field = $\frac{\text{charge density}}{\varepsilon_0}$

Resistivity $\rho = \frac{1}{\sigma}$



$|\text{induced emf}| = \frac{\omega B l^2}{2}$

Energy density of an electric field

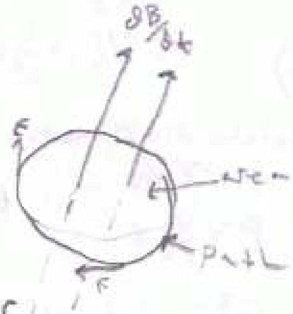
$$E = \frac{1}{2} \varepsilon_0 \cdot |\varepsilon|^2$$

Maxwell's equations

① Gauss's law for E $\int_{\text{closed surface}} E \cdot \partial A = \frac{Q_{\text{net within}}}{\varepsilon_0}$

② Gauss's law for B $\int_{\text{closed surface}} B \cdot \partial A = 0$

③ Faraday's law $\oint_{\text{path } \Gamma} E \cdot \partial r = \frac{-\partial}{\partial t} \int_{\substack{\text{surface bounded} \\ \text{by A}}} B \cdot \partial A = -\frac{\partial \Phi_{mag}}{\partial t}$



④ Ampere-mawell law $\oint_{\text{path } \Gamma} B \cdot \partial r = \mu_0 I + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \int_{\text{surface whose}} E \cdot \partial A$